

Maths for Computing

Tutorial 7

1. A busy airport sees 1500 takeoffs per day. Prove that there are two planes that must take off within a minute of each other.
2. The set M consists of 9 positive integers, none of which has a prime divisor larger than 6. Prove that M has two elements whose product is the square of an integer.
3. Let H be a 10-element set of 2-digit positive integers. Prove that H has two disjoint subsets A and B so that the sum of the elements of A is equal to the sum of the elements of B .
4. There are four heaps of stones in our backyard. We rearrange them into five heaps. Prove that at least two stones are placed into a smaller heap.
5. A host invites n couples to a party. She wants to ask a subset of the $2n$ guests to give a speech, but she does not want to ask both members of any couple to give speeches. In how many ways can she proceed?
6. In how many ways can the elements of $[n]$ be permuted so that the sum of every two consecutive elements in the permutation is odd?
7. A traveling agent has to visit four cities, each of them five times. In how many different ways can he do this if he is not allowed to start and finish in the same city?
8. A student in physics needs to spend five days in a laboratory during her last semester of studies. After each day in the lab, she needs to spend at least six days in her office to analyse the data before she can return to the lab. After the last day in the lab, she needs ten days to complete her report that is due at the end of the last day of the semester. In how many ways can she do this if we assume that the semester is 105 days long?